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ANALYTICAL MODELS FOR THE STRESS-STRAIN BEHAVIOR OF CONFINED AND UNCONFINED CONCRETE MASONRY

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Abstract

Two mathematical models are presented to represent the idealized stress-strain behavior of both confined and unconfined grouted concrete masonry in compression. The models are based on experimental data from recent prism tests, and account for varying types of confinement reinforcing. The "Acceptable Fit" model is a simple model suggested for use in design. The "Best Fit" model is more complex but more accurate, and is appropriate for use in computer analyses.

Introduction

A detailed knowledge of the stress-strain behavior of concrete masonry is necessary to predict the performance of masonry structures responding inelastically under seismic loads. The most basic measure of the stress-strain behavior of masonry is the uniaxial compressive stress-strain curves obtained from prism tests. Several important parameters of masonry behavior can be derived from stress-strain curves: the maximum compressive stress, strain at maximum stress, the maximum usable strain, and the shape of the falling branch. These may then be directly related to the maximum available ductility of masonry elements. In this paper, mathematical models of masonry stress-strain curves will be developed based on recent experimental data for both unconfined and confined concrete masonry in uniaxial

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compression. Models will be presented for five different types of confinement reinforcement.

Two different mathematical models -- a simple model and a more complex model -- will be developed for experimental stress-strain curves from unreinforced prisms and several varieties of confined prisms. The simple model requires an engineer to supply only one shape parameter and two material parameters to completely describe the stress-strain curve. The more complex model, more appropriate for research or computer analysis, requires three shape parameters in addition to two material parameters.

The work reported here is drawn from the analytical phase of a research effort designed to study the effect of confinement reinforcement on the behavior of concrete masonry prisms and shear walls. In the first experimental phase of the research, 106 concrete masonry prisms were tested, including unreinforced prisms, reinforced unconfined prisms, and confined prisms using seven different confinement schemes. Experimental tests of shear walls with confinement reinforcement are currently in progress. All experiments have been designed to complement data from the TCCMAR U.S./Japan Coordinated Program for Masonry Research. For a complete description of the experimental program, the reader is referred to references [1,2 and 3].

Analytical Models

Two analytical models are developed. The first, called the Acceptble Fit model, is developed with the requirement that it be as simple as practically possible. A second model, the Best Fit model, is developed to provide a more accurate fit to the experimental data at the cost of some complexity. After choosing the desired mathematical functions for the models, the values of the parameters in each model are obtained for the given experimental data using least squares curve fitting techniques.

In order to simplify the fitting functions, the stress-strain curves are divided into two regions, namely the rising branch (the portion up to and including the ultimate stress) and the falling branch (the portion following the ultimate stress). These regions are illustrated in Figure 1. In the following sections, the mathematical functions for each region of both models are treated individually. Notation is given at the end of the paper.

The Acceptable Fit Model

The simplest approximation of the rising branch of the stress-strain curve is given by a straight line from the origin up to the peak stress and corresponding strain, and can be written:

$$f_m(\varepsilon_m) = f'_m (\varepsilon_m/\varepsilon_u) \quad (1)$$

This straight line function can be defined with only the two material parameters f'_m and ε_u.

The falling branch is modeled by an exponential function of the form:

$$f_m(\varepsilon_m) = f'_m B e^{(-A*\varepsilon m)} \quad (2)$$

The parameter B can be elliminated by enforcing the boundary condition that $f_m=f'_m$ at $\varepsilon_m=\varepsilon_u$, thus:

$$f_m(\varepsilon_m) = f'_m e^{(-A(\varepsilon m-\varepsilon u))} \quad (3)$$

For the Acceptable Fit model, the structural engineer need only supply values for one shape parameter (A) and two material parameters (f'_m and ε_u) to completely define the stress-strain curve.

The Best Fit Model

The rising branch of the Best Fit model is defined by a second degree polynomial of the form:

$$f_m(\varepsilon_m) = A\varepsilon_m{}^2 + B\varepsilon_m \quad (4)$$

Upon applying the boundary condition that $f_m=f'_m$ at $\varepsilon_m=\varepsilon_u$, and defining a new parameter $C\equiv A/f'_m$, the equation can be written in terms of one shape parameter and two material parameters:

$$f_m(\varepsilon_m) = f'_m [C\varepsilon_m{}^2 + ((1/\varepsilon_u) - C\varepsilon_u{}^2)\varepsilon_m] \quad (5)$$

The falling branch of the Best Fit model assumes an exponential function of the form:

$$f_m(\varepsilon_m) = B + Ce^{(-E\varepsilon m)} \quad (6)$$

Again enforcing the boundary condition that $f_m=f'_m$ at $\varepsilon_m=\varepsilon_u$, and defining D = B/f'm the equation for the falling branch can be written:

$$f_m(\varepsilon_m) = f'_m [D + (1-D)e^{(-E(\varepsilon m-\varepsilon u))}] \quad (7)$$

Thus there are three shape parameters (C, D and E) and two material parameters (f'_m and ε_u) that must be defined in order to completely describe the stress-strain curve using the Best Fit model.

Presentation of Results

Following development of the analytical models, the chosen mathematical functions were used to fit curves to the experimental data. The curves were fit to averaged experimental data obtained by averageing the results of four to five prism tests from the same series. Table 1 describes the prism series for which curves were fit. Table 2 gives the values of the shape parameters calculated for each model and each prism series in the least squares analysis.

The results of a curve fit for unreinforced, grouted concrete masonry prisms are illustrated in Figures 2 and 3 for the Acceptable Fit and the Best Fit respectively. In Figure 2, a comparison of the experimental and analytical curves on the rising branch shows that the straight line idealization of the Acceptable Fit model underestimates the stress for any given strain level up to the strain at ultimate stress. Figure 3 shows that the Best Fit model nearly coincides with the experimental data on the rising branch. Similarly, on the falling branch the Best Fit model provides a very close approximation to the experimental data, while the Acceptable Fit is less accurate. However, considering the simplicity of the Acceptable Fit model, its accuracy is satisfactory for purposes of design.

The results of the curve fit analyses on six other prism series are illustrated in Figures 4-9.

Discussion of Results

The variation in the curve fitting parameters listed in Table 2 reveals trends in the data that can be used to compare different confinement schemes. For the Best Fit model, the parameter "C" for the rising branch decreases with increasing compressive strength, and shows that confinement reinforcement provides a small increase in compressive strength. In the falling branch, parameters "D" and "E" both tend to decrease as the slope of the falling branch decreases and the curve flattens out. The trend in the parameters shows the positive effect of confinement on the stress-strain curves.

A comparison of the curve parameters for the various confinement schemes represented in Table 2 reveals that any confinement scheme (SP1, SP2, OWM1 or OWM2) is superior to the minimum confinement reinforcement recommended in Section 2412 of the 1988 UBC (UBC1), i.e., #3 bars at 8 inches on center. While the UBC Type 1 confinement does yield some improvement in the shape of the descending branch of the stress-strain curve, other confinement schemes offer significantly better results. Furthermore, a comparison between Open Wire Mesh (OWM) and Spiral (SP) confinement schemes shows that the OWM2 confinement is superior to the spiral.

Conclusions

Two mathematical models have been developed to model the stress-strain behavior of confined and unconfined

TABLE 1

TYPES OF PRISMS USED FOR ANALYTICAL STUDY

NO.	CONFINEMENT TYPE *, **	I.D. CODE
1	No Steel	URM
2	Vertical Steel Only	UCO
3	Vertical Steel + #3 Ties @ 8" o.c.	UBC1
4	Vertical Steel + Spiral Type 1	SP1
5	Vertical Steel + Spiral Type 2	SP2
6	Vertical Steel + Open Wire Mesh Type 1	OWM1
7	Vertical Stell + Open Wire Mesh Type 2	OWM2

* Type 1: Steel Volume Approximately Equivalent to UBC minimum (#3 @ 8" o.c.).

** Type 2: 2 times Type 1 Confinement Steel.

TABLE 2

VARIOUS PARAMETERS INVOLVED IN CURVE FITTING

TYPE OF CONFINE-MENT.	RISING BRANCH	FALLING BRANCH		
	BEST FIT	ACP. FIT	BEST FIT	
	C	A	D	E
URM	-146900	472.5	0.110	613.8
UCO	-176700	437.1	0.131	574.6
UBC1	-175400	240.6	0.201	516.2
SP1	-179700	159.6	0.243	309.8
SP2	-169650	148.0	0.327	435.5
OWM1	-143100	161.7	0.216	314.4
OWM2	-186300	110.8	0.185	165.1

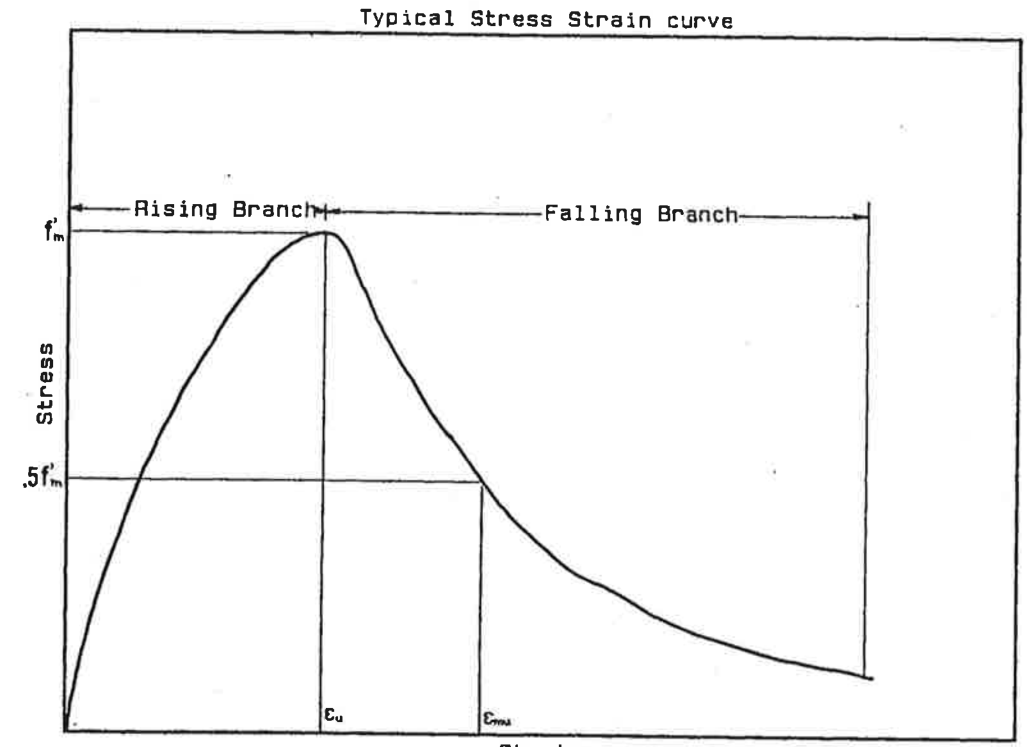

Figure 1

UNREINFORCED CONCRETE MASONRY(URM)
EXPERIMENTAL AND IDEALIZED STRESS STRAIN CURVES

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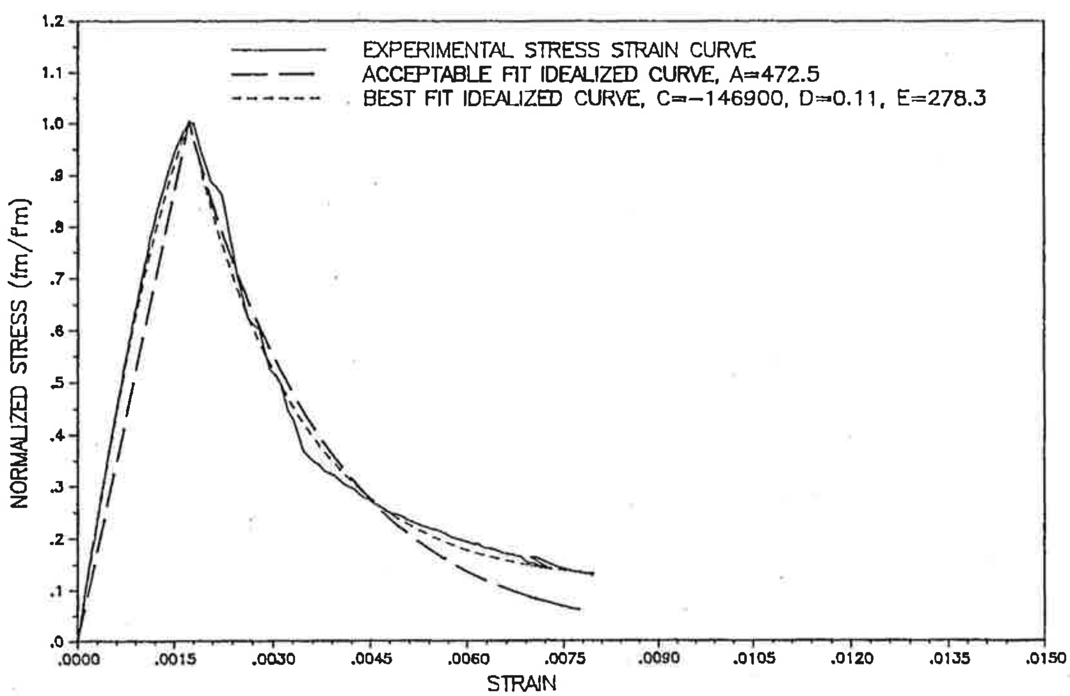

FIGURE 2.0 IDEALIZED STRESS STRAIN CURVES FOR URM

UNCONFINED CONCRETE MASONRY(UCO) EXPERIMENTAL AND IDEALIZED STRESS STRAIN CURVES	ENGLEKIRK & HART INC. ATKINSON–NOLAND & ASSOC.

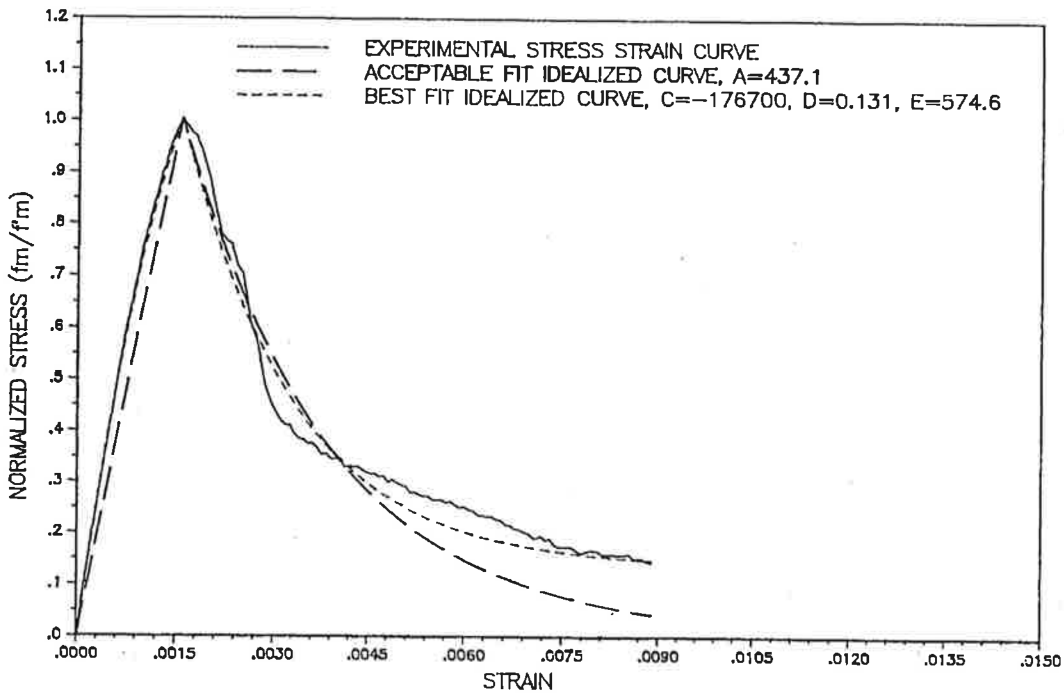

FIGURE 3.0 IDEALIZED STRESS STRAIN CURVES FOR UCO

UBC TYPE 1 (#3 @ 8" o.c) EXPERIMENTAL AND IDEALIZED STRESS STRAIN CURVES	ENGLEKIRK & HART INC. ATKINSON–NOLAND & ASSOC.

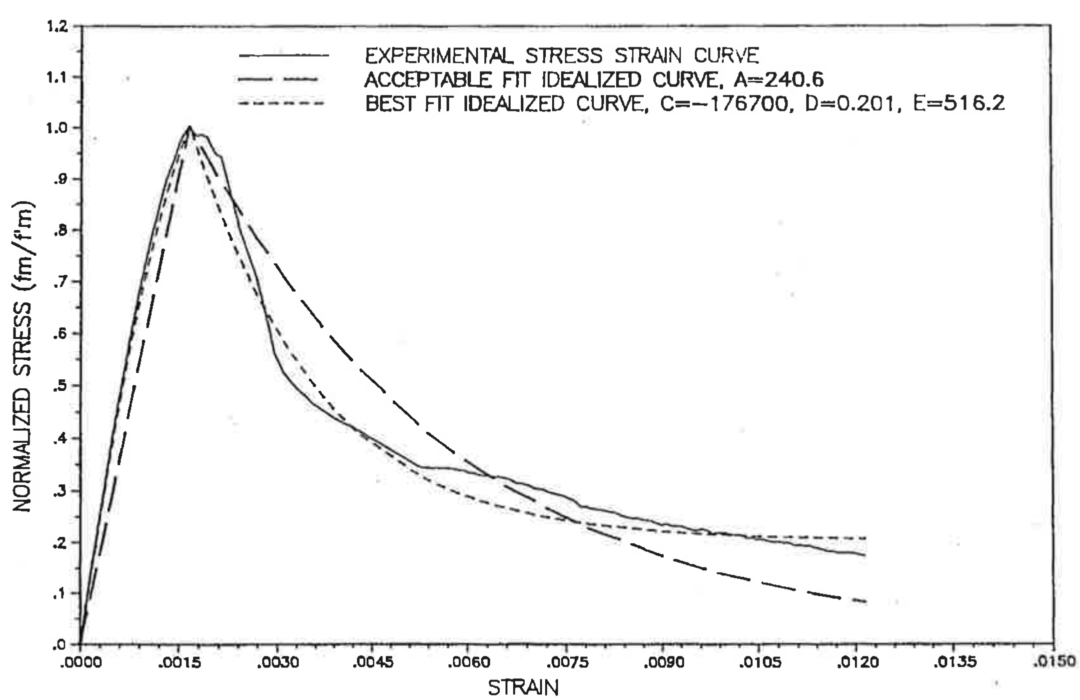

FIGURE 4.0 IDEALIZED STRESS STRAIN CURVES FOR UBC1

SPIRAL TYPE 1 (#4 WIRE @ 2" PITCH)
EXPERIMENTAL AND IDEALIZED STRESS STRAIN CURVES

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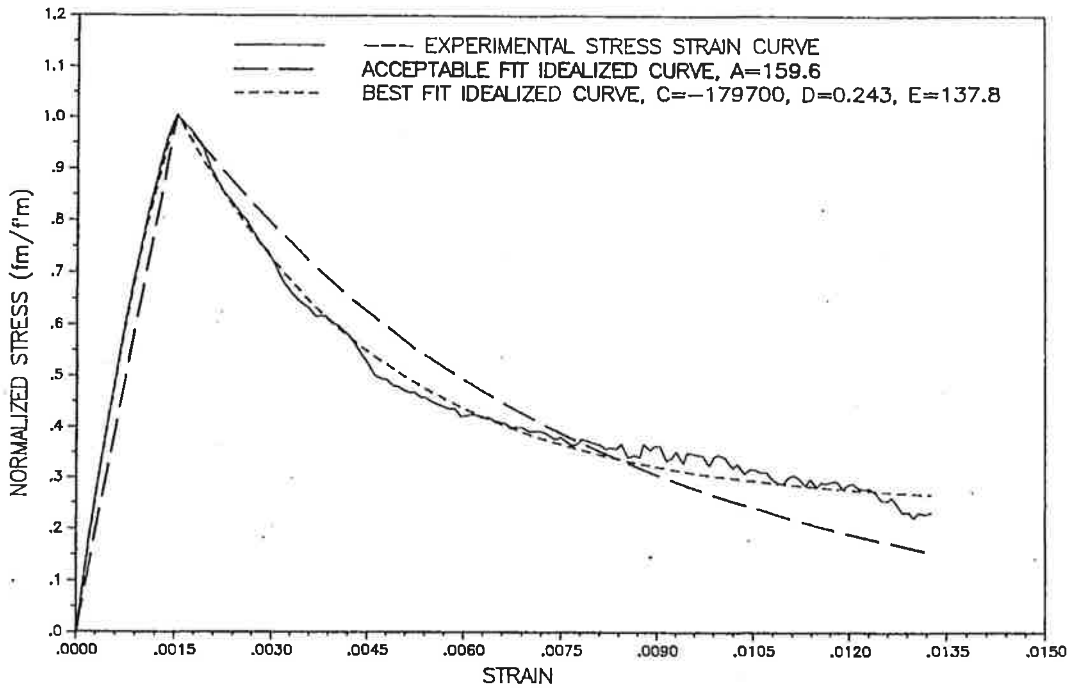

FIGURE 5.0 IDEALIZED STRESS STRAIN CURVES FOR SP1

SPIRAL TYPE 2 (#12 WIRE @ 2" PITCH)
EXPERIMENTAL AND IDEALIZED STRESS STRAIN CURVES

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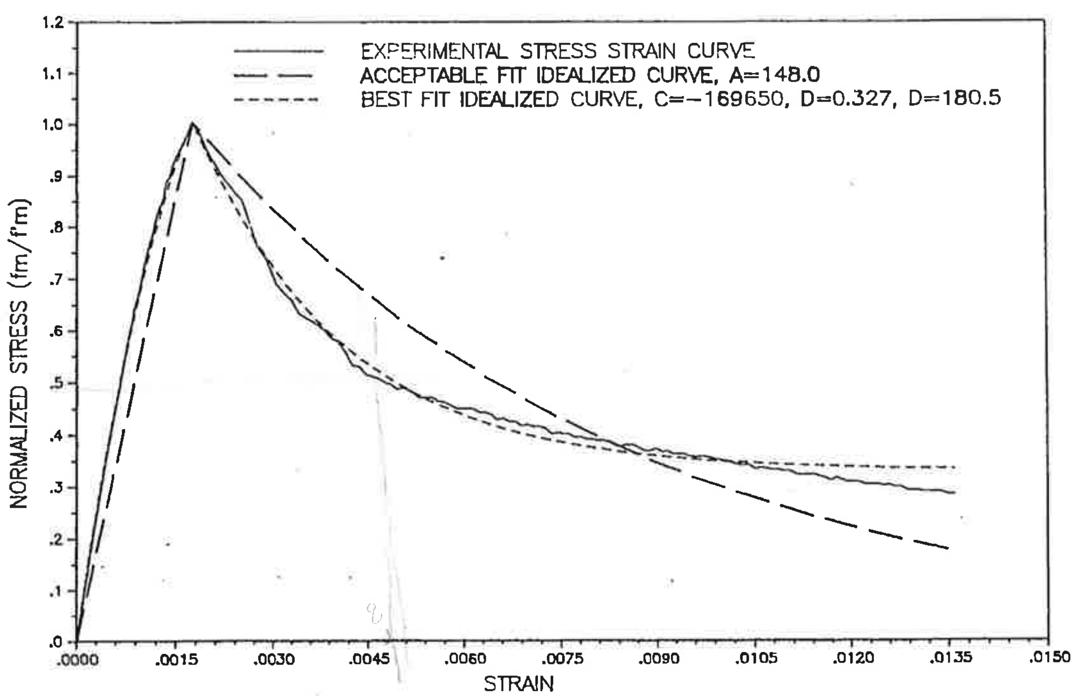

FIGURE 6.0 IDEALIZED STRESS STRAIN CURVES FOR SP2

OPEN WIRE MESH TYPE 1 (#9 WIRE)
EXPERIMENTAL AND IDEALIZED STRESS STRAIN CURVES

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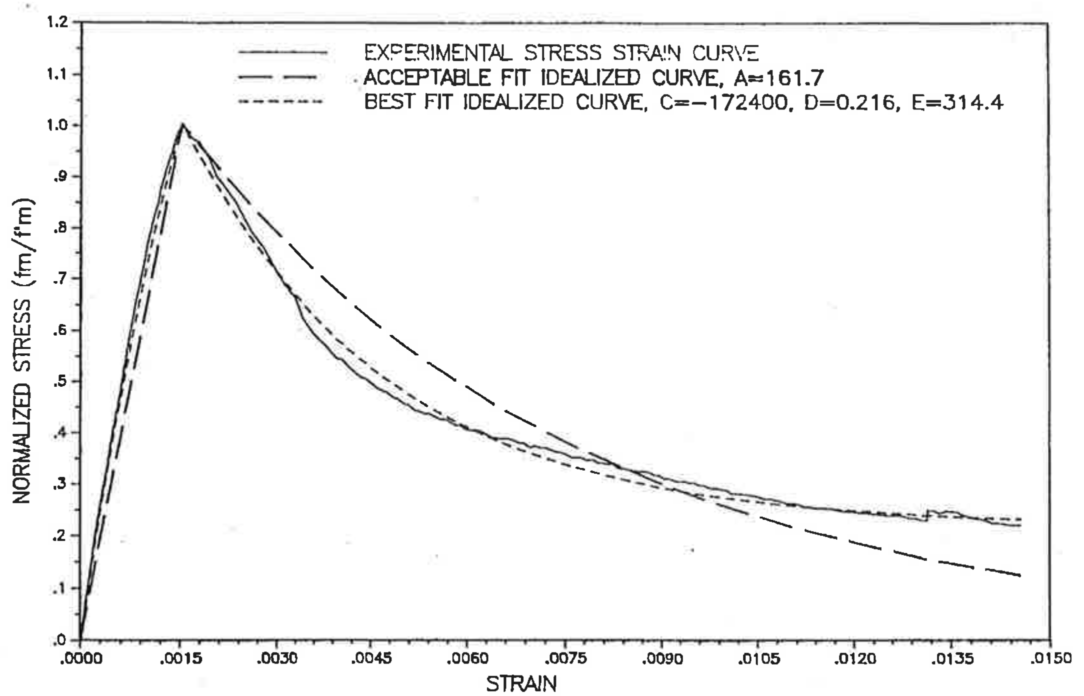

FIGURE 7.0 IDEALIZED STRESS STRAIN CURVES FOR OWM1

OPEN WIRE MESH TYPE 2 (3/16" WIRE)
EXPERIMENTAL AND IDEALIZED STRESS STRAIN CURVES

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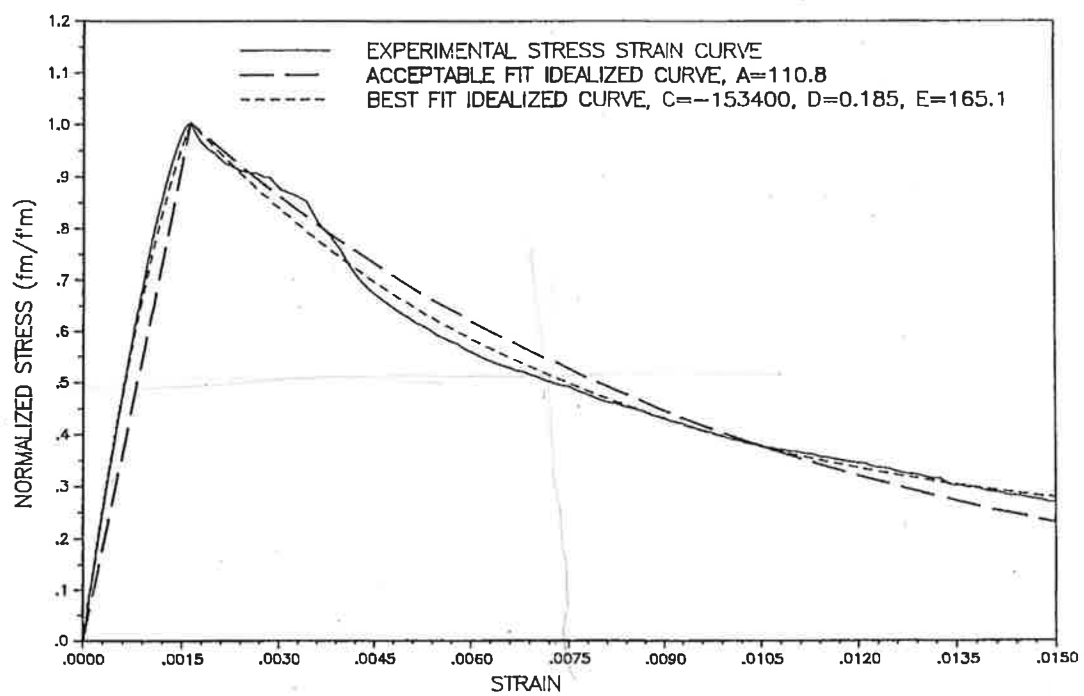

FIGURE 8.0 IDEALIZED STRESS STRAIN CURVES FOR OWM2

masonry in compression: the Acceptable Fit model and the Best Fit model. The Acceptable Fit model uses a linear rising branch and a second order falling branch, and requires an engineer to specify only one shape parameter and two material parameters (f'_m and ε_u). It is simple, and yet models stress-strain curves well enough to be suitable for design. The Best Fit model is more complex, and requires three shape parameters to be specified in addition to the two material parameters. It models stress-strain curves very well, and is appropriate for detailed analysis using micro-computers.

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Notation

A-E = Curve fitting parameters
f_m = Concrete masonry compressive stress
f'_m = Maximum compressive stress of concrete masonry
ε_m = Concrete masonry compressive strain
ε_u = Strain corresponding to maximum masonry compressive stress